calibration constants must be adjusted in proportion to the rate of test loading. A test engineering challenge for the future is the use of an ARE model to obtain a fatigue evaluation prior to the completed component fatigue test. For example, if one needs to determine fatigue strengths of various complex hole geometries subjected to a biaxial complex load pattern, and if these geometries can be machined or molded from an ARE, it is possible that fatigue results can be obtained with ARE. Preliminary test investigations in this area, using ARE bars in a smooth and a hole configuration, have yielded promising results (Fig. 8), but many problems such as notch sensitivity, aging, brittleness, and model similitude yet require detailed investigation.

Conclusions

This ARE model technique constitutes an important laboratory tool. Development of an ARE with an average 15-min modulus of over 1×10^6 psi and an average ultimate strength of 7000 psi has provided the necessary structural material. An ARE model provides not only true perspective visualization for the designer after he has completed preliminary drawings, but also a fast, inexpensive means of assessing structural integrity and for adding or removing materials to optimize the configuration. If loads and moduli are correctly proportioned, the resultant stress distribution will be identical to those of its metal counterpart. These molded models are easy to machine, easy to rework, and simple to load.

The primary disadvantage of the material is inherent in its viscoelastic properties. Further studies of the complex characteristics and variations in materials and mixes, and further development on conversion of model results to accurate final-configuration strains, are needed. However, for a general understanding of stress distribution and concentrations, and a quick comparison of configurations and geometry, the process is considered developed.

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Stress Concentration Factors for Bonded Lap Joints

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This paper is concerned with the stress analysis by the finite element method of a bonded, single, lap joint. Since the adhesive layer is of primary importance, the stresses occurring in it are presented. A modified version of the well-known Wilson stress analysis program is used for the case of plane stress. The total length beyond the lap is considered long compared to the lap length. Stress concentrations as functions of dimensionless, geometric and material parameters are presented. For a given load σ, the important maximum shearing stress concentration, $\tau_{\max}/\sigma$, and tearing stress concentration $\sigma_{\max}/\sigma$, are plotted as functions of l/t and η/t for different values of E/E_A, where l is the lap length, η is the adhesive thickness, t is the thickness of the material being bonded, and E and E_A are the moduli of elasticity for the adherend and the adhesive, respectively.

Nomenclature

σ = applied stress
τ_{xy} = shear stress
σ_y = tearing stress
$\tau_{\max}$ = maximum shear stress
$\sigma_{\max}$ = maximum tearing stress
L = length of each end section
l = length of lap
t = adherend thickness
η = adhesive thickness
E = adherend elastic modulus
E_A = adhesive elastic modulus
ν = Poisson's ratio

Introduction

Of considerable importance in the design of a bonded joint is the decision as to which type of joint would be most advantageous. Of the more common types, the lap joint and its variations are presently the most practical and most studied. The lap joint can be basically one of two types, the single lap joint, with one bondline, and the double lap joint, with two bondlines.

The double lap joint in tension is the simpler type to analyze since there is no bending incurred during deformation, and considerable work has been done in this area.[1,4,10]

Of more practical importance is the single lap joint, which has likewise received considerable attention. The first reliable treatment of the problem was given by Goland and Reissner[5] who performed an analytical stress analysis of cemented lap joints in 1944. However, the simplifications necessary to make the analytical solution possible have restricted the results. Basically, Goland and Reissner considered two extreme cases. One was based on the assumption that most of the deformation occurred in the adhesive layer, as in metal to metal bonding; the other treated an inflexible adhesive layer, as in a wood to wood joint. A few years later Cornell[3] studied a simplification of Goland and Reissner's problem by replacing the adhesive with a system of tension and shear springs. Thus, his analysis neglected the Poisson's ratio effect, and the stresses parallel to the joint. Subsequently, many papers have been published, all with analytical or experimental simplifications.[2,6-9] Little work has been done in this area using numerical techniques, and to the

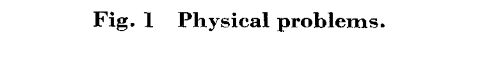

Fig. 1 Physical problems.

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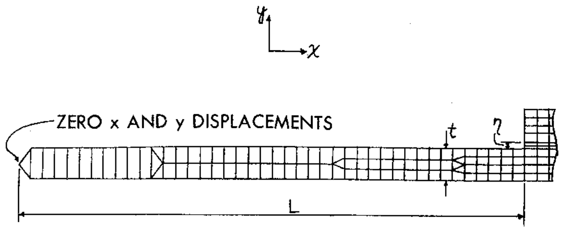


Fig. 2 Finite element grid for end section (same grid for both ends).

Table 1 Comparison of stress concentrations

Joint parameters			Maximum tearing stress concentrations	
E/E_A	l/t	η/t	Finite element method	Goland and Reissner
10	5	0.06	1.43	1.44
50	5	0.06	0.65	0.64
100	5	0.06	0.59	0.63

author's knowledge the finite element method has never before been used to attack this problem.[12]

Solution of Problem

The physical problem, corresponding to the numerical solution given in this paper, is presented in Fig. 1. Solution was by the finite element method,[13] which was implemented by a modified form of the stress analysis program prepared by E. L. Wilson at the University of California, Berkeley.[11] Input into the program was the location of 497 nodal points, and description of 394 elements by indicating the nodal points and material properties for that element. To make possible the analyzing of many lap joints with different geometric and material properties, a program was written to generate the input to the stress analysis program, primarily the location of the nodal points. The locating of nodal points was accomplished by dividing the configuration into three parts, the section of the lap joints, and the sections to each side of the lap. Each part was given a grid of mostly quadrilateral "finite elements" by dividing the length and thickness into equal segments. The end sections were each divided into 42 equal segments along its length, and from 1 to 4 equal segments through its thickness, as shown in Fig. 2. The lap joint section was separated into 20 equal parts along its length and 10 equal segments through the total thickness of the bondline and two adherends, as illustrated in Fig. 3. It is of some significance that the adhesive layer was divided into two equal layers through its thickness, giving an indication of the variation of stresses in the direction normal to the bondline. Thus, nodal points were located as a function of the length and thickness of each section, that is, in terms of the geometric parameters of the configuration. Additionally, the material parameters of the adhesive and adherend were read into the program. Therefore, by simply changing a few input cards, a single lap joint with any material and geometric properties could be analyzed.

The Wilson program is based on a linear displacement function within each element, which yields a constant strain and therefore constant stress over each triangular element, with the quadrilateral elements composed of four triangular elements. The constant stress is considered to act at the centroid of each element, resulting in a discontinuous approximation to the stress field. Therefore, to evaluate the stress acting at any particular point a linear interpolation or extrapolation from centroidal values of stresses is necessary. Since there were two elements through the thickness of the adhesive, a linear extrapolation was used to determine the maximum stresses acting at that section. The maximum shear stress concentration was assumed to be at a distance from the free edge of one-half the element width, which is the location of the centroids for the two end elements. Therefore, the maximum shearing stress was calculated with only one linear extrapolation. However, the maximum tearing stress concentration occurs at the free edge, so an additional extrapolation was necessary from the element centroid to the edge of the element.

The geometric and material properties considered are expected to include any problem to be encountered in practice. Poisson's ratio was considered to be the same for each material, with $\nu = 0.30$; but this assumption was not considered too restrictive since Poisson's ratio varies over a small range compared to the range of other properties. An additional reason for neglecting this parameter was the scarcity of information available concerning Poisson's ratio for adhesives.[2] The dimensionless elastic modulus parameter E/E_A ranges from 0.1 to 1000, where E is the elastic modulus of the adherend and E_A is that of the adhesive. The lap length, l, end section length, L, adherend thickness, t, and bondline or adhesive thickness, η, were all treated as independent variables, with the length of the end sections, L, being considered arbitrarily large, $L/t = 100$. A check was made to insure that the L/t ratio, for $L/t > 100$, did not affect the results significantly. In fact the stress concentration for $L/t = 50$ did not differ appreciably from those for $L/t = 100$. The lengths were nondimensionalized with respect to the adherend thickness, yielding dimensionless parameters, η/t and l/t. Figure 5 shows typical distributions of tearing and shear stresses in the bondline. For a given load σ, the maximum tearing stress concentration, $\sigma_{\max}/\sigma$, and the maximum shearing stress concentration, $\tau_{\max}/\sigma$, were plotted as functions of the dimensionless parameters discussed above; these are shown in Fig. 5.

Conclusions

Since there are no complete analytical solutions to this problem, the comparison of the results obtained by the finite element method to other solutions was restricted. However, one important comparison was made with the limited cases

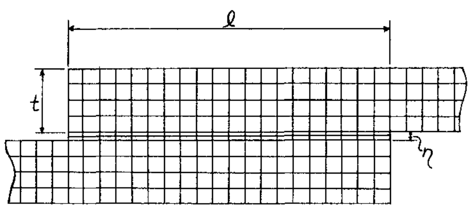


Fig. 3 Finite element grid for lap joint section.

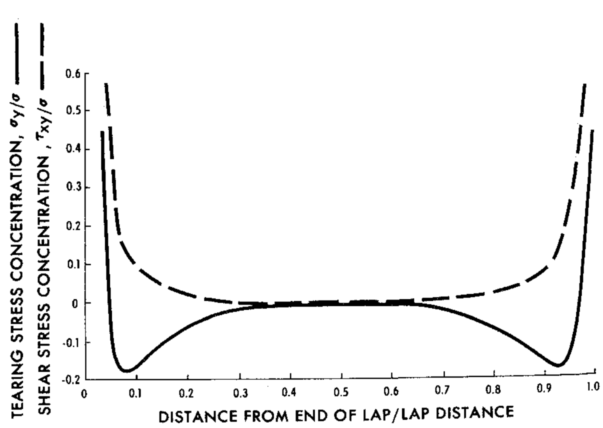
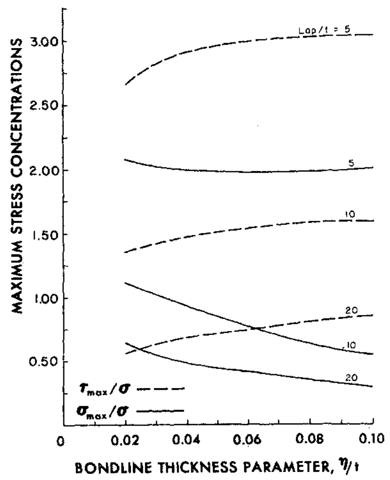
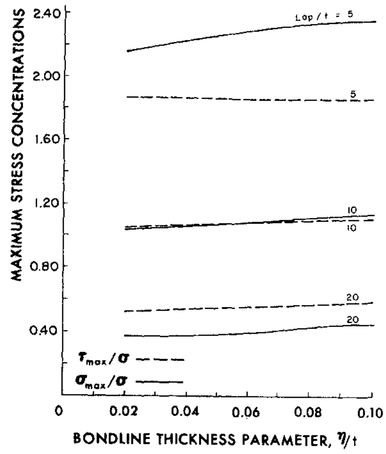


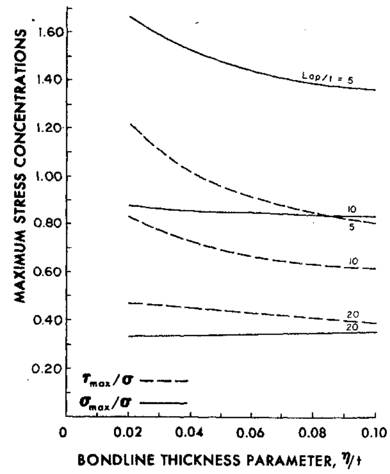
Fig. 4 Variation in stresses at center of bondline.



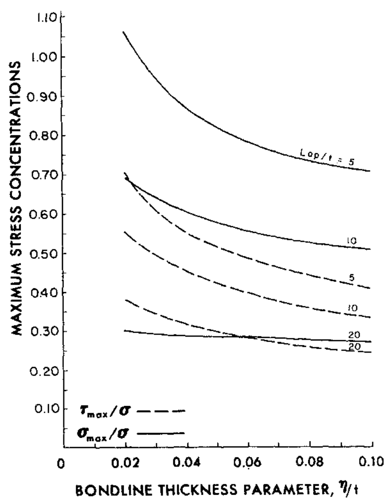
a) Ratio of $E/E_A = 0.1$.



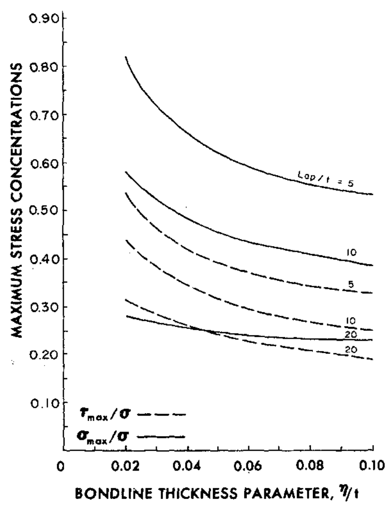
b) Ratio of $E/E_A = 1$.



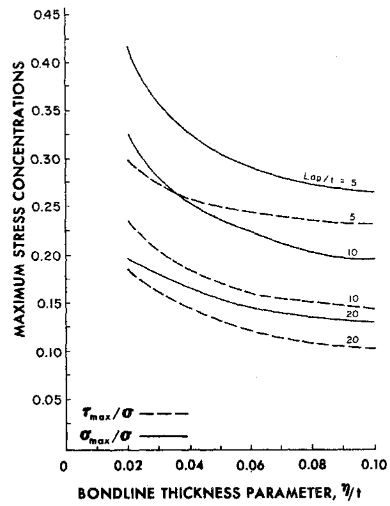
c) Ratio of $E/E_A = 10$.



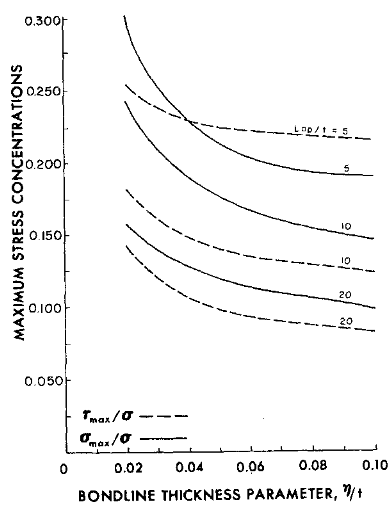
d) Ratio of $E/E_A = 50$.



e) Ratio of $E/E_A = 100$.



f) Ratio of $E/E_A = 500$.



g) Ratio of $E/E_A = 1000$.

Fig. 5 Maximum stress concentrations vs bondline thickness parameter for elastic modulus.

considered by Goland and Reissner.[5] It should be emphasized that the solution given in this paper was obtained by a plane stress analysis, whereas Goland and Reissner made the assumption of plane strain for their solution. The stress concentrations obtained by each method were compared at three values of the geometric and material parameters, as given in Table 1. The results of the two approaches compare very favorably.

A comment regarding accuracy is, perhaps, in order. The finite element model used in the analysis is an approximation to the physical problem, yielding results which are approximations to the actual stresses which would occur. It is general knowledge that a large aspect ratio for the finite element quadrilaterals should be avoided. The extreme case presented, $\eta/t = 0.02$, $l/t = 20$, has an aspect ratio of 100:1. A check was made by recomputing the stresses using an aspect ratio of 40:1, with essentially no difference in the shearing stress distribution and hence no difference in the shearing stress concentration factor. The tearing stress distributions computed in the two cases differed only in the region near the reentrant corner where high stress gradients exist. Extrapolating the centroidal values of the stress to the surface requires considerable judgement. In both cases the curves are nearly vertical, but the smaller aspect ratio unquestionably yields larger tearing stress concentration factors. The authors hope that the curves presented in this paper will be of value in the design of bonded joints.

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Dynamic Performance Characteristics of Mixed and Unmixed Turbofan Engines

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Mixed and unmixed type turbofans, having identical rotating components and thermodynamic cycles, are compared in terms of transient characteristics and steady-state off-design operation. Effects of step changes in fuel flow rate, nozzle area, inlet pressure, ambient temperature and air bleed are illustrated on both the fan and compressor operating maps. Results show significant differences in the dynamic behavior of the mixed and unmixed turbofan engines without the influence of an engine control. Off-design steady-state operating points are also different. Both effects are more pronounced in the low-pressure fan than in the high-pressure compressor. Such characteristics are important in the design of a compatible engine control unit.

Introduction

THE turbofan engine in its mixed and unmixed form is the most common engine in modern civil aircraft. Mixed augmented turbofans are finding many applications as the power plant for supersonic military aircraft. Thermodynamic design and turbofan cycle optimization have been studied extensively. Pearson[1] and others have explained the advantages of the mixed turbofan engine. References 2 and 3 summarize the methods of thrust augmentation by energy transfer and mixing between the primary and the secondary air flows. In general, the turbofan cycle is superior to the basic turbojet engine. However, the performance advantages of the mixed to unmixed turbofan is only a few percent.

Engine dynamic characteristics, important in design of compatible control units, are quite different for turbojets and turbofans. Also, significant differences between the flow dynamics of mixed and unmixed turbofan can be recognized. In this paper the steady-state off-design and transient behavior of these two types of turbofan engines are discussed. Both models used are two-spool fixed geometry engines with identical rotating components and identical thermodynamic cycle parameters. The analytical method of Refs. 4 and 7 is applied to investigate the transient behavior with step changes in fuel flow rate, nozzle area and other disturbances which are characteristic in modern engine applications. Reference 5 shows a similar engine comparison for acceleration and deceleration under command input and in closed loop.

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Engine and Control Inputs

The construction of both turbofans investigated are compared in Fig. 1, with the upper half illustrating the mixed